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Economic and Financial Consulting and Expert Testimony

**DELOS Damages and Valuation MOOC:
Module 2.2
Income Approach – Calculating the Discount Rate**

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Important Caveats

- The views expressed in this presentation are solely those of the author, who is responsible for the content, and do not necessarily represent the views of Cornerstone Research.
- Real-life cases are complicated – expert experience, judgment and discretion are required in addressing many of the issues covered by this presentation.

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Introduction

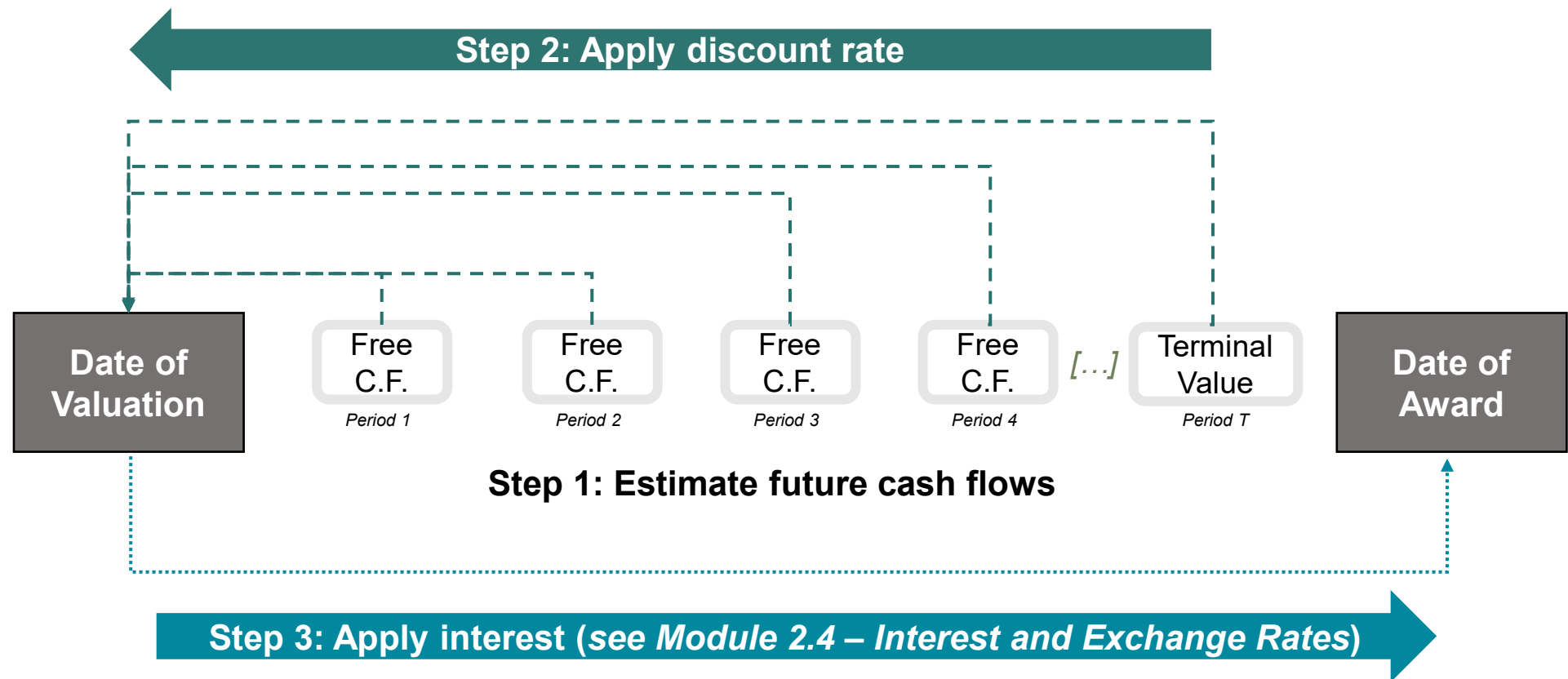
Relationship to Other Modules

- Module 1.6 – Income Approach:
 - Introduced the discounted cash flow (“DCF”) method of valuation—to value a company or project (a “firm”), discount the firm’s expected future cash flows back to the date of valuation at a rate which reflects both the time value of money and the riskiness of those future cash flows
- Module 2.1 – Income Approach (Advanced):
 - Addressed various complexities in DCF valuation

Recap of Discounted Cash Flow (“DCF”) Valuation

- Basic setting
 - Seeking to value a “firm” that is financed by some combination of debt and equity
- Have previously seen two possible approaches:
 - Value company/project in its entirety by discounting free cash flows to the “firm” at the weighted average cost of capital (“WACC”)
 - Value equity in company/project by discounting free cash flows to equity at the cost of equity
 - For all-equity firm, two approaches are the same ($WACC = \text{cost of equity}$)
- Important, and often overlooked fact—for either approach to be valid, need to assume that proportions of debt and equity in the firm’s capital structure will remain constant through time

Estimation and discount of future cash flows



Stable Capital Structure

- Proportions of debt and equity remaining constant through time means that if, e.g., firm performs better than expected and value increases, it will raise more debt and/or repurchase equity
- Stylized version of reality (in practice, changes to a firm's capital structure will be “lumpy”) but, in most circumstances, making this assumption will not create too many problems
 - However, in some cases, it can be very far from reality:
 - e.g., in private equity space, common for firm to have a proportionately large amount of debt initially, to be repaid quickly out of cash generated by operations
 - In this case, use of WACC would be extremely questionable—much more appropriate to use a variant of DCF valuation known as adjusted present value (“APV”)
- For remainder of module, will maintain stable capital structure assumption

Recap: WACC

- WACC equation:

$$WACC = (1 - W)r_E + Wr_D(1 - T)$$

where

- W is leverage ratio (proportion of firm's capital structure that is accounted for by debt), so that $(1 - W)$ is proportion of firm's capital structure that is accounted for by equity
- r_E is the cost of equity (the expected return that investors in the firm's equity require from their investment)
- r_D is the cost of debt (the expected return that investors in the firm's debt require from their investment)
- T is the corporate tax rate
 - Multiply by $(1 - T)$ because debt interest is tax-deductible expense for firm, tax saving reduces cost of debt financing from firm's perspective.

2 Leverage Ratio

Determining the Leverage Ratio

- By definition, what should go into the WACC calculation is the forward-looking leverage ratio i.e., the proportion of the firm's capital structure that will be accounted for by debt
 - Remember that this is assumed to be constant through time
- Key point—for the purposes of a WACC calculation, the leverage ratio is defined in terms of market values, not book values
 - In many cases (but not always), book value of debt will be a reasonable proxy for market value—this is unlikely to be the case for equity



Determining the Leverage Ratio, cont'd.

- If firm is publicly traded (so that market value of equity can be observed), can estimate forward looking leverage by calculating historical leverage ratio over some period (e.g., past five years) and “averaging”
- If firm is not publicly traded, identify a set of comparable companies that are publicly traded, calculate the historic leverage ratios of these companies (again over some period of time) and average
 - In fact, even if firm is publicly traded, may want to use comparable companies as a robustness check
- Note—may need to apply some judgment rather than taking a simple average:
 - Are there any obvious outliers?
 - Are there any obvious trends?

3

Cost Of Equity

Capital Asset Pricing Model (“CAPM”)



- CAPM equation: cost of equity (r_E) given by

$$r_E = r_f + \beta(E[r_M] - r_f)$$

where

- r_f is the risk-free rate [compensation for time value of money]
- β is the firm's equity beta [measure of risk inherent in an investment in the firm's equity]
- $E[r_M]$ is the expected return on the overall market, so that $E[r_M] - r_f$ is the expected market risk premium (“EMRP”) [compensation per unit of risk]
 - EMRP also known as the “equity risk premium” or “ERP”

Risk-Free Rate



- Three key questions:
 - What is meant by a risk-free rate?
 - In what currency should the risk-free rate be measured?
 - What is the appropriate maturity for the risk-free rate input to the CAPM equation?

Concept of Risk-Free Rate

- Risk-free investment: one where investor knows with absolute certainty what they will receive in the way of a cash payoff and when; risk-free rate is rate of return on this investment
 - e.g., if current price of a security paying \$121 with absolute certainty two years from now (and nothing else) is \$100, the (two-year) risk-free rate is 10% (since $100 = 121/1.10^2$)
- For an investment to be risk-free, needs to offer a fixed cash payoff and needs to be default-free
 - Typically, this means a debt security issued by a sovereign government, although not all sovereign debt is default-free e.g., Euro denominated debt issued by Italy



Currency of Risk-Free Rate

- Is a debt security issued by the Indian government and denominated in rupees risk-free?
 - To a reasonable approximation, yes—government can always print more of its own currency
- Is a debt security issued by the Indian government and denominated in US dollars risk-free?



No—such a security carries a (potentially significant) default risk

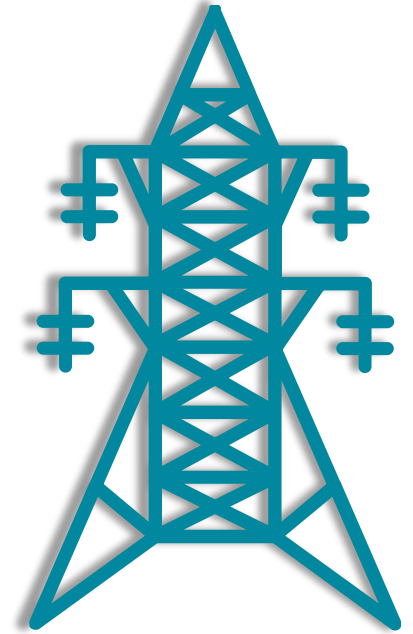
- Implication—the risk-free rate for a CAPM calculation needs to match the currency of the cash flows being discounted

Currency of Risk-Free Rate, cont'd.

- Example—power plant in India:
 - Expected future cash flows denominated in Indian rupees, use rate of return on a rupee-denominated debt security issued by the Indian government



- Expected future cash flows denominated in US dollars, use rate of return on a US dollar-denominated debt security issued by the US government



Maturity of Risk-Free Rate

- Cost of equity r_E is input to $WACC$ which is used to discount all expected future cash flows—so what is appropriate maturity of risk-free rate for CAPM calculation of r_E (one-year, two-year, etc.)?
- No single right answer—general acceptance that the appropriate maturity is one that “matches” the average maturity of the expected future cash flows you are looking to discount, but this is a somewhat vague prescription
 - In practice, many experts will use either a five-year or a ten-year rate



EMRP (ERP)

- In principle, easy to define: “the difference between the expected return on....the overall market less the return offered on a risk-free investment” [Holthausen and Zmijewski, p.309]—however:
 - “Overall market”—generally understood to be a market capitalization-weighted average of “all” listed equity securities, but does “all” mean US, World, country in which firm is located?
 - What is the relevant time horizon—are we interested in what is expected to happen over the next year, the next five years, etc.?
 - What is the relevant currency—e.g., if we have decided [more later as to why] that the overall market is the world market, and we are looking to value a power plant in India, are we interested in the rupee-denominated EMRP or the EMRP denominated in some other currency?

EMRP (ERP), cont'd.

- In practice, these questions are typically ignored—implicit assumption is that there is a single EMRP that is independent of choice of overall market, time horizon, currency
- So question becomes—how do we estimate this EMRP?
 - Historical approach—what happened in the past (on average) can be expected to happen in the future
 - Implied cost of capital approach
 - Survey evidence
 - e.g., Graham and Harvey survey of US CFOs
 - “Over the next 10 years, I expect the average annual S&P 500 return will be...”
 - “During the next year, I expect the S&P 500 return will be...”



Estimation of EMRP: Historical Approach

- Decide on proxy for overall market (S&P 500, FTSE 100, CAC 40, etc.)
- Obtain return data on this proxy over some historical period
- Obtain risk-free returns over the same period
 - Maturity of risk-free assets [T-Bills, T-Bonds]?
- Calculate realized “excess return” (market *minus* risk-free) over each period
- Calculate average (arithmetic or geometric)

Estimation of EMRP: Historical Approach, cont'd.

Results will differ across markets, over different time periods, by choice of risk-free benchmark, and by choice of arithmetic vs. geometric average

- US 1900 to 2016, bills, arithmetic: 7.4%
- US 1900 to 2016, bonds, geometric: 4.3%
 - Premium relative to bills almost always higher than premium relative to bonds (maturity premium), arithmetic average always higher than geometric average
- US 1926 to 2011, bills, arithmetic: 8.2%
- France 1900 to 2016, bills, arithmetic: 8.7%
- World 1900 to 2016 (USD), US bills, arithmetic: 5.6%



Estimation of EMRP: Historical Approach, cont'd.

- Issues with historical approach
 - Structural changes over time: declining risk → lower required return → capital gain → higher historical return/risk premium
 - Survivorship bias: observed historical returns can be used as a proxy for expected return conditional on surviving—unconditional expected return (taking into account possibility of non-survival) is lower

[Credit Suisse Global Investment Returns Yearbook, published annually, provides a very careful discussion of how historical experience can be used to inform future expectations]

Estimation of EMRP: Summary

- Different approaches will give (potentially very) different results
- Experts will often take results from a variety of sources and use judgment as to the most appropriate EMRP to input to the CAPM—consistency is important

Note: You can find a further discussion on the Estimation of EMRP: Implied Cost of Capital Approach in Section 6: “Supporting Material (Cost of Equity)”

Beta

- Systematic vs non-systematic risk:
 - Systematic risk cannot be diversified away whereas non-systematic risk can
 - Investors should only be compensated for bearing systematic risk
- Beta is measure of how much systematic risk is inherent in an investment in the firm's equity and reflects:
 - Expected volatility of the investment relative to the overall market
 - Extent to which returns on the investment are expected to be correlated with returns on the overall market
- Note that beta more accurately described as “equity beta”—if returns on the firm's debt are risky (because of e.g., default risk), then it is meaningful to talk about the firm's “debt beta”

Estimating Equity Beta

- Suppose equity in firm has been publicly traded for a number of years
- Can estimate historical equity beta using a statistical technique known as linear regression
- Note: due to mean reversion/estimation error, may adjust estimate towards 1
 - e.g., Bloomberg:

$$\beta_{adjusted} = 0.67 \times \beta_{estimated} + 0.33 \times 1$$

Estimating Equity Beta, cont'd.

- Questions to address:
 - Domestic or world index?
 - Domestic reflects implicit assumption that capital markets are “segmented” while world reflects implicit assumption that capital markets are “integrated”
 - Estimation period?
 - Longer period implies more data (more precise estimate) but nature of firm activities may change over time
 - Periodicity of returns (daily, weekly, monthly)?
 - More frequent data implies more data (more precise estimate) but may be more susceptible to “market microstructure” effects (e.g., “thin” trading)
- In practice, should always examine sensitivity of results to these choices

Other Source of Beta Estimates

- Website of Professor Aswath Damodaran (<https://pages.stern.nyu.edu/~adamodar/>) contains beta estimates for a range of industries and markets (US, Europe, Japan, etc.) – many experts will use this data “as is”, but important not to treat as black box
 - Ensure that methodology is fully understood
 - Ensure that any limitations e.g., instability through time are acknowledged

Note: You can find further discussions on Beta Levered and Unlevered, the Use of Historical Equity Beta in CAPM, Estimating Equity Beta, and BARRA Betas in Section 6: “Supporting Material (Cost of Equity)”

4 Cost Of Debt

What is the Cost of Debt?

- By definition, the cost of debt for a given firm is the expected return that investors require—taking risk into account—from an investment in that firm's debt
- General observation (applies to any financial investment)—if it is fairly priced, the expected return offered by the investment is equal to the required expected return
- Implication: cost of debt r_D is what you need to discount expected debt cash flows at in order to get back to the value of debt
- Need to be careful here: expected debt cash flow is not the same as promised debt cash flow



Yield to Maturity and Expected Default Losses

- Typical to estimate cost of debt by looking at the promised yield to maturity (YTM) on the firm's debt (this is what you need to discount promised debt cash flows at in order to get back to the value of debt)
- Because promised debt cash flows are greater than expected debt cash flows, YTM will overstate r_D

$$YTM - r_D = EDL \text{ (expected default loss)}$$

YTM, r_D , and EDL

- Example

- One year debt, face value 1,000, coupon rate 9%

- Probability of default 4%—in case of default, no interest payment, 60% recovery on principal

- Expected debt cash flow is

$$0.96 \times 1,000 \times 1.09 + 0.04 \times [0.6 \times 1,000] = 1,070.40$$

- Current value of debt 973.09

- Cost of debt is 10%: $973.09 = \frac{1,070.40}{1.10}$

- YTM is 12%: $973.09 = \frac{1,090}{1.12}$

- EDL is 12% - 10% = 2%

Estimating the Cost of Debt

- Approach 1:
 - Estimate YTM (requires that value of firm's debt is observable)
 - Subtract estimate of EDL (see e.g., Holthausen and Zmijewski....)
- Approach 2:
 - Use CAPM
 - Typically, very difficult to estimate debt beta directly (limited data)
 - “Workaround”
 - Assume that $r_D = YTM$ ($EDL = 0$) for AA rated debt
 - For lower rated debt [note very little AAA rated corporate debt], estimate r_D as follows:
$$r_D = YTM \text{ on generic AA debt} + \text{incremental beta} \times EMRP$$

5 Other Issues

Various Risk Premia

- Different experts will have different views as to whether to add (either to the cost of equity or to the WACC) a:
 - Country risk premium
 - Size premium
 - Firm-specific risk premium

Country Risk Premium

- A commonly made argument is the following:
 - “Investments in certain, particularly emerging market, countries are riskier than investments in countries such as the United States—as such, the discount rate should be increased to reflect this additional risk”
- When this argument is made, the increase in the discount rate—the country risk premium—is typically estimated as the sovereign spread (difference between yield on US treasuries and yield on US-denominated sovereign bonds issued by the country in question).
 - When not available, reference is often made to Damodaran website (but see earlier caveats)



Country Risk Premium, cont'd.

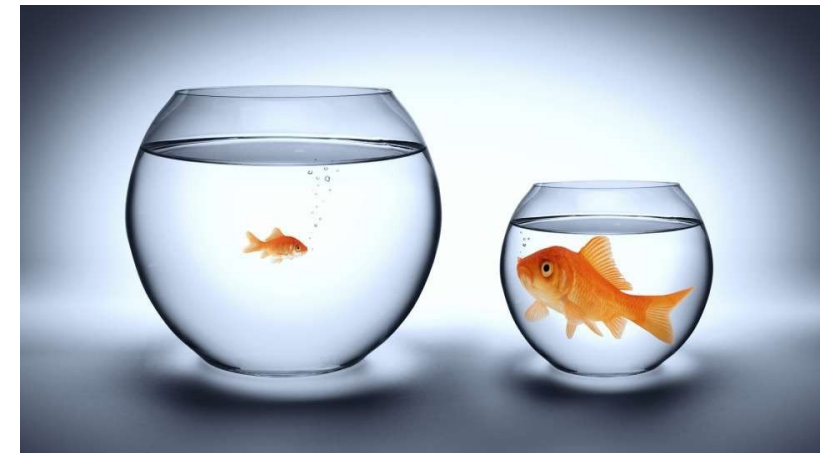
- Controversial topic
 - Are international investments riskier than domestic investments?



- If yes, are the additional risks systematic or non-systematic?
 - If systematic, adjusting discount rate is appropriate – but is the sovereign spread the correct measure of the adjustment?
 - If non-systematic, correct approach (difficult to implement in practice) is to adjust expected cash flows downwards

Size Premium

- Another commonly made argument is the following:
 - “Investments in small firms are riskier than investments in otherwise identical large firms”
- Support for this argument is based on academic research that demonstrates that small firms have historically, after adjusting for risk (measured by beta), yielded higher average returns than large firms
- However, recent research has questioned whether the “small firm effect” is disappearing over time and whether it exists in all markets
 - Another controversial topic



Firm Specific Risk Premium

- Some experts will argue that “investors should be compensated for bearing non-systematic risk” and will therefore add a risk premium to cover firm specific risks
- This has no academic support and is entirely contrary to the notion that such risks can be diversified away and should therefore not command a risk premium

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Supporting Material (Cost of Equity)

Estimation of EMRP: Implied Cost of Capital Approach

- On a stock-by-stock basis, use:
 - Current free cash flows
 - Analyst or model-based forecasts of short/intermediate-term cash flow growth
 - Long-term industry growth rates/forecast economy growth rates

to generate estimates of expected future cash flows and determine the discount rate that yields the current stock price

- Average (using market capitalization) to get expected return on overall market
- Subtract current risk-free rate to get estimate of EMRP

Estimation of EMRP: Implied Cost of Capital Approach, cont'd.

- Advantage: is explicitly forward-looking
- Disadvantage: is heavily dependent on forecasts of future growth

Beta: Levered and Unlevered

- Important to distinguish between a firm's levered beta β_L (its beta given its anticipated [constant] leverage ratio L) and its unlevered beta β_U (the beta it would have with a leverage ratio of zero i.e., all-equity financed)
- Relevant for CAPM: the β in $r_E = r_f + \beta(E[r_M] - r_f)$ is the levered beta β_L ; common to write β_E rather than β_L
- Relationship between levered and unlevered betas:

$$\beta_E = \beta_U + \left[\left(1 - \frac{T_C r_D}{1 + r_D} \right) \left(\frac{L}{1 - L} \right) \right] (\beta_U - \beta_D) \text{ [Equation 1]}$$

where β_D is the beta of the firm's debt; often approximated as

$$\beta_E = \beta_U + \left(\frac{L}{1 - L} \right) (\beta_U - \beta_D) \text{ [Equation 2]}$$

Beta: Levered and Unlevered, cont'd.

- With this approximation, can “invert” this and write

$$\beta_U = (1 - L)\beta_E + L\beta_D \text{ [Equation 3]}$$

- Note 1: β_U is “fundamental” and reflects the risk of firm’s operating activities, while β_E reflects both this risk and the additional risk introduced by having debt in the capital structure (leverage “magnifies” risk to investors in the firm’s equity)
- Note 2: Common to assume $\beta_D = 0$, but note that this implies either that debt is risk-free (no default risk) or that any default risk is uncorrelated with general economic conditions

Use of Historical Equity Beta in CAPM

- If the anticipated leverage ratio is the same as the historical leverage ratio, can input the firm's estimated historical equity beta (possibly adjusted) to the CAPM to estimate the firm's cost of equity
- If the anticipated leverage ratio is different from the historical leverage ratio:
 - Use the estimated historical equity beta (possibly adjusted), the historical leverage ratio, and an estimate of β_D in Equation 3 to generate an estimate of β_U
 - Use this estimate of β_U , together with the anticipated leverage ratio and the estimate of β_D in Equation 2 to generate an estimate of the “new” equity beta
- it is the latter that is input to the CAPM to estimate the firm's cost of equity

Estimating Equity Beta (II)

- Suppose equity in firm is not publicly traded
- In this case:
 - Identify a set of comparable companies that are publicly traded
 - Estimate the historical equity beta for each comparable company (possibly adjust)
 - Use **Equation 3** to convert each estimated historical equity beta to an estimated historical unlevered beta
 - Will need information on historical leverage ratio and an estimate of the debt beta for each company
 - Take an average (mean, median, precision-weighted) of these estimated historical unlevered betas to obtain an estimate of the firm's unlevered beta β_U

Estimating Equity Beta (II), cont'd

- Use this estimate of β_U , together with the firm's anticipated leverage ratio and an estimate of the firm's debt beta in **Equation 2** to generate an estimate of the firm's equity beta
- Input this to the CAPM to estimate the firm's cost of equity

BARRA Betas

- “*Predicted beta*, the beta BARRA derives from its risk model, is a forecast of the stock’s sensitivity to the market. It is also known as *fundamental beta*, because it is derived from fundamental risk factors. In the BARRA model, these risk factors include 13 attributes—such as size, yield, and price/earnings ratio—plus industry exposure allocated across a maximum of 6 of 55 industry groups. Because we reestimate these risk factors monthly, the predicted beta reflects changes in the company’s underlying risk structure in a timely manner. BARRA programs use predicted beta rather than historical beta because it is a better forecast of market sensitivity.”
- Main disadvantage: lack of transparency as to exactly how it is calculated—has been raised as an issue by various courts and tribunals.

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